IB MYP YEAR 5

YEAR 10 Mathematics

Assessment #4

AP, GP, LOGARITHM, EXPONENTIAL, BEST FIT

Name: KWOK Chun Hei (10 Joy)

Teacher: **Ms. Li, Mr. So & Mr. Wong**

Date of task: **Friday, 22nd February, 2013** Time allowed: **2 lessons**

Student's **Performance** in Different **Criterion**			
A	8	**D**	6

INSTRUCTIONS:

Read the **rubric** carefully because that is how you will be graded.
Read the instructions for all questions carefully.
Show all work, steps and proper units.
Write your answers on the lined paper/graph paper provided.
GDCs are allowed.
Ask the teacher for scrap paper, but any work on the scrap paper will **NOT** be marked.
Allowed to use calculators.
Allowed to use **non-electronic dictionary**.

Criterion A: KNOWLEDGE AND UNDERSTANDING

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	
1–2 Simple	The student manipulates the data correctly, but **without** reaching a full model.	The student **generally** makes appropriate deductions when solving **simple** problems in **familiar** contexts.	(0-8)
3–4 Complex	The student is able to come up with a **basic** model and make simple **predictions** based on the model.	The student **generally** makes appropriate deductions when solving **more complex** problems in **familiar** contexts.	**Teacher's Final Grade**
5–6 Challenging	The student is able to come up with an **appropriate** model and **explain** the processes by which the model was created.	The student **generally** makes **appropriate** deductions when solving **challenging** problems in a **variety** of **familiar** contexts.	
7–8 Unfamiliar	The student comes up with an appropriate model which fits the initial data and then comes up with an **unfamiliar model** which also fits the **new data**.	The student **consistently** makes **appropriate** deductions when solving **challenging** problems in a **variety** of contexts including **unfamiliar** situations.	(0-8)

Criterion D: REFLECTIONS & EVALUATIONS

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not submit a poster. Or, the students submit work with unacceptable quality.	The student does not reach a standard described by any of the descriptors given below.	
1–2 Real Life	Student talks about real life changes in Hong Kong which cause changes in the growth of rubbish.	The student **attempts to explain** whether his or her results make sense in the context of the problem. The student **attempts to describe** the importance of his or her findings in connection to real life.	(0-6)
3–4 Degree of Accuracy	In considering the percentage error, student can justify which model is more accurate in predicting waste in 2009-11. Percentage error must be attempted to achieve a level 4.	The student **correctly but briefly explains** whether his or her results make sense in the context of the problem. The student **describes the importance** of his or her findings in connection to real life where appropriate. The student **attempts to justify** the degree of accuracy of his or her results where appropriate.	**Teacher's Final Grade**
5–6 Improve-ments	Student thinks critically about units and the improvements of the second model from the first model. Percentage error is used to catch mistakes if any.	The student **critically explains** whether his or her results make sense in the context of the problem. The student provides a **detailed explanation** of the importance of his or her findings in connection to real life. The student **justifies the degree of accuracy** of his or her results where appropriate. The student suggests improvements to his or her method where appropriate.	(0-6)

HOW DOES TRASH IN THE TKO LANDFILL GROW?

The government proposes expanding the Tsang Kwan O landfill (opened at the start of 2000). You have gathered the following data:

Year	2001	2002	2003	2004	2005	2006	2007	2008
m^3/day	1,749	1,903	2,040	2,274	2,549	2,645	2,811	2,940

Table 1: Commercial & Industrial Solid Waste in Hong Kong[1]

You suspect there is a predictable pattern in the growth of waste dumped at the site.

1. **Find** a mathematical model that appropriately describes the growth. **Explain** the processes by which the model is created?

considering the growth is in arithmetic progression,

the common difference $= \frac{154+137+234+275+96+166+129}{7} = 170$ (corr. to 3 sig.fig)

$\therefore T_n = 1749 + (n-1)170 = 1749 + 170n - 170 = 170n + 1579$

using the equation,

$T_1 = 1749$
$T_2 = 1919$
$T_3 = 2089$
$T_4 = 2259$
$T_5 = 2429$
$T_6 = 2599$
$T_7 = 2769$
$T_8 = 2939$

% error (corr. to 3 sig. fig.)

T_1: 0%
T_2: 0.84%
T_3: 2.40%
T_4: -0.66%
T_5: -4.71%
T_6: -1.74%
T_7: -1.49%
T_8: -0.03%

Average % error of the equation is -0.68% (corr. to 3 sig fig)

$\therefore$ the percentage error of the mathematical model of $T_n = 1749 + (n-1)170$ is very close to 0, this model appropriately describes the growth.

1 The Government of the Hong Kong Special Administrative Region, "Monitoring of Solid Waste in Hong Kong," Disposal of Solid Waste at Landfills: Commercial & Industrial Waste (plate 2.3), 2000-2010, viewed January 13, 2013, http://www.wastereduction.gov.hk/en_html/assistancewizard/waste_red_sat.htm

2. By using the model you acquired, predict which year the amount of waste will be **more than 3200 m^3/day**.

$T_n = 1749 + (n-1)170$

$1749 + (n-1)170 > 3200$

$(n-1)170 > 1451$

$n-1 > \frac{1451}{170}$

$n > \frac{1451}{170} + 1$

$> \frac{1621}{170}$

> 9.54 (corr. to 3 sig. fig.)

∴ In 2009, the amount of waste will be more than 3,200 m^3/day.

3. Predict the amount of waste OVER THE ENTIRE YEAR dumped in **2009**, **2010** and **2011** based on your model.

2009:

$[1749 + (9-1)170]365$

$= [1749 + 8(170)]365$

$= (1749 + 1360)365$

$= 3109(365)$

$= 1,134,785$

∴ In 2009, the total amount of waste dumped is $1,134,785\ m^3$.

2010:

$[1749 + (10-1)170]365$

$= [1749 + 9(170)]365$

$= (1749 + 1530)365$

$= 3279(365)$

$= 1,196,835$

∴ In 2010, the total amount of waste dumped is $1,196,835\ m^3$.

2011:

$[1749 + (11-1)170]365$

$= [1749 + 10(170)]365$

$= (1749 + 1700)365$

$= 3449(365)$

$= 1258885$

∴ In 2011, the total amount of waste dumped is $1,258,885\ m^3$.

4. The actual amount of waste dumped in 2009, 2010 and 2011 were 1,089,160 m^3/year, 1,087,335 m^3/year and 1,103,395 m^3/year. Using **percentage errors** or any other valid method, come up with the degree of accuracy of your predictions in question 3. **How accurate was your model?**

2009:

$$\%\ \text{error} = \frac{1,134,785 - 1,089,160}{1,089,160} \times 100\%$$

$$= \frac{45625}{1,089,160} \times 100\%$$

$$= \frac{125}{2984} \times 100\%$$

$= 4.19\%$ (corr. to 3 sig. fig.)

∴ With the percentage error of 4.19%, which is slightly far from 0%, this mathematical model is rather inaccurate

2010:

$$\%\ \text{error} = \frac{1,196,835 - 1,087,335}{1,087,335} \times 100\%$$

$$= \frac{109,500}{1,087,335} \times 100\%$$

$$= \frac{100}{993} \times 100\%$$

$= 10.1\%$ (corr. to 3 sig. fig.)

∴ With the percentage error of 10.1%, which is far from 0%, this mathematical model is inaccurate.

2011:

$$\%\ \text{error} = \frac{1,258,885 - 1,103,395}{1,103,395} \times 100\%$$

$$= \frac{155,490}{1,103,395} \times 100\%$$

$$= \frac{426}{3023} \times 100\%$$

$= 14.1\%$ (corr. to 3 sig. fig.)

∴ With the percentage error of 14.1%, which is really far from 0%, this mathematical model is inaccurate at all.

5. What changes in **Hong Kong in 2009** could account for any change in the pattern of waste growth?

In 2009, Hong Kong has confronted the financial crisis, which affected the economic development in Hong Kong. Thus capital constructions as well as constructions of commercial and industrial buildings were paused. Due to this effect, Hong Kong has produced less commercial and industrial waste, for instance, timber, bricks, glass and concrete, which could account for the change of waste growth. However, on the other hand, Hong Kong has experienced the spreading of the H1N1 influenza in 2009. As the influenza is a threatening disease, citizens are advised to wear facial masks and clean their floats or office workstation regularly. Hence, citizens consume more facial masks and bottles of cleanser or alcohol, which also produce more wastes than expected. Therefore, both the financial crisis and the H1N1 influenza could account changes of the increase or decrease in waste growth.

6. Use your calculator to come up with a **new** model (not linear or exponential). Write down the model below.

$y = ax+b$ $y = ab^x$

By comparing possible mathematical models of quadratics, cubic, quartic and logarithmic, logarithmic appropriately presents the growth.

This is determined by the correlation coefficient (r) shown on the GDC, in which the correlation coefficient of quadratic regression is N/A, cubic regression is N/A, quartic regression is N/A, but logarithmic regression is 0.9577501...

Considering the logarithmic growth of $y = a + b\ln x$,

that $a = 1571.64197$

$b = 597.652195$

∴ The new mathematical model is $T_n = 1571 + 598\ln n$

7. Use **both** your **original** and **new** models to predict the amount of waste generated in **2013**.

Original:

$T_n = 1,749 + (n-1)\,170$

$T_{13} = 1,749 + (13-1)\,170$

$= 1749 + 12(170)$

$= 1,749 + 2040$

$= 3,789$

Total waste generated in 2013

$= 3,789 \times 365$

$= 1,382,985$

∴ It is predicted the amount of waste generated in 2013 is $1,382,985\ m^3$.

New:

$T_n = 1,571 + 598 \ln n$

$T_{13} = 1,571 + 598 \ln 13$

$= 3,104$

Total waste generated in 2013

$= 3,104 \times 365$

$= 1,132,960$

∴ It is predicted the amount of waste generated in 2013 is $1,132,960\ m^3$.

8. How do the results in Q7 compare? Explain the differences in your results. Why are they important?

The results of the original and new model compare as the predicted amount of wastes generated using linear is $250,025\ m^3$ more than using logarithm. This happens because the linear equation has a gradient of +170, which is much steeper than the flatter curve logarithmic equations. They are important as it is key for the Hong Kong government know when will the landfill be full so that they can start planning the expansion of the Tseung Kwan O landfill, meaning by how much land has to be reclaimed to meet the large demand.

THE END